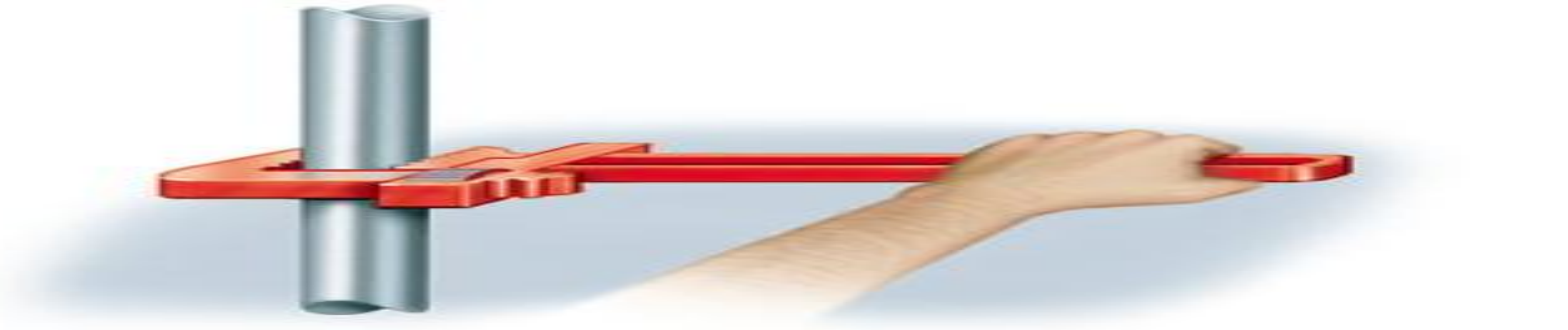
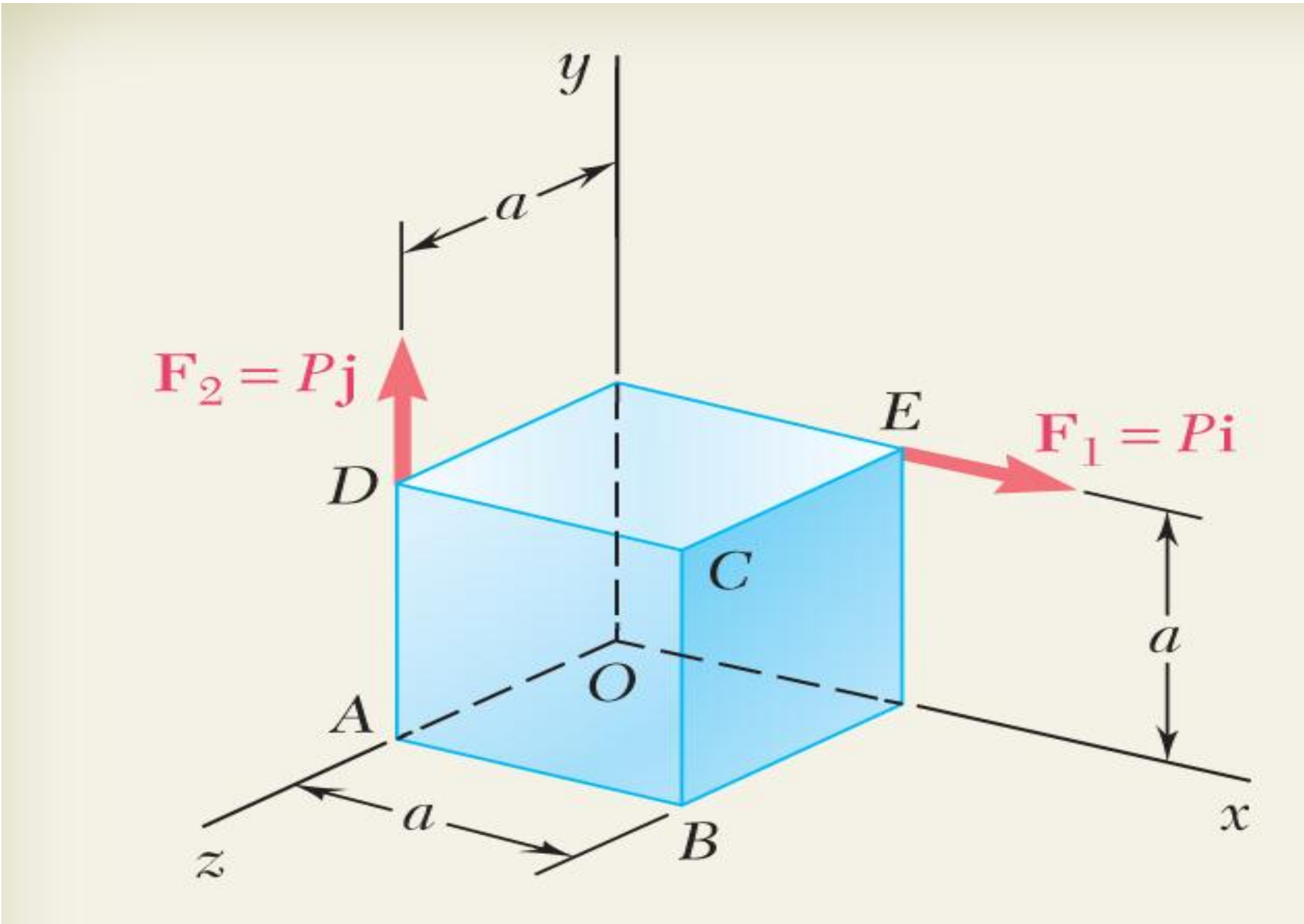
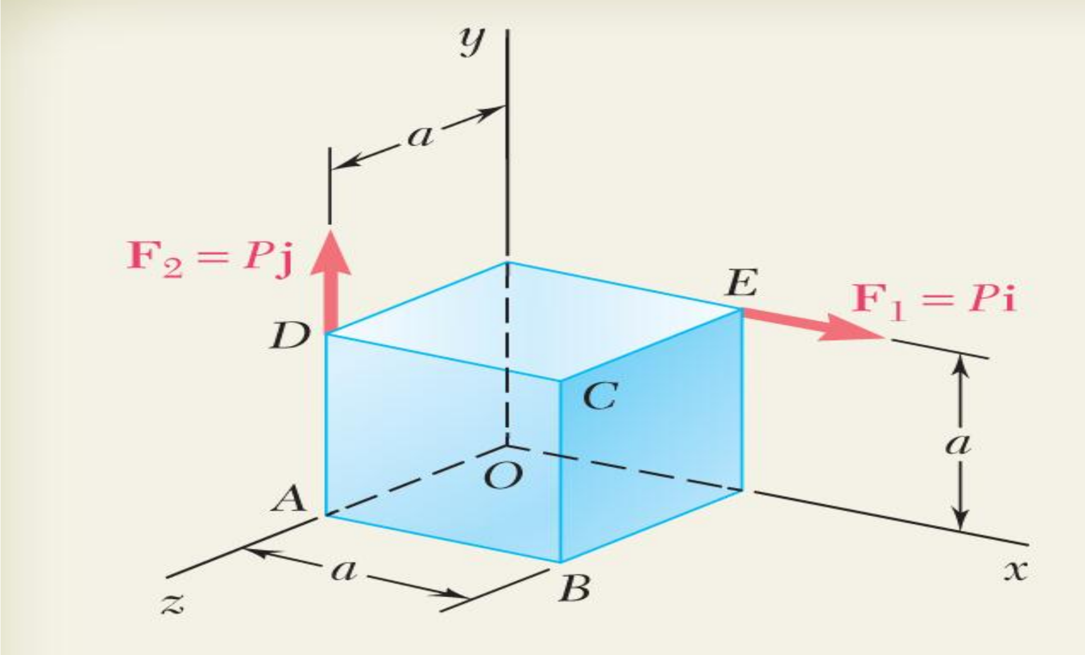


4 Force System Resultants

د محمود عبد المولى

Replace the two forces by a force and couple at point o





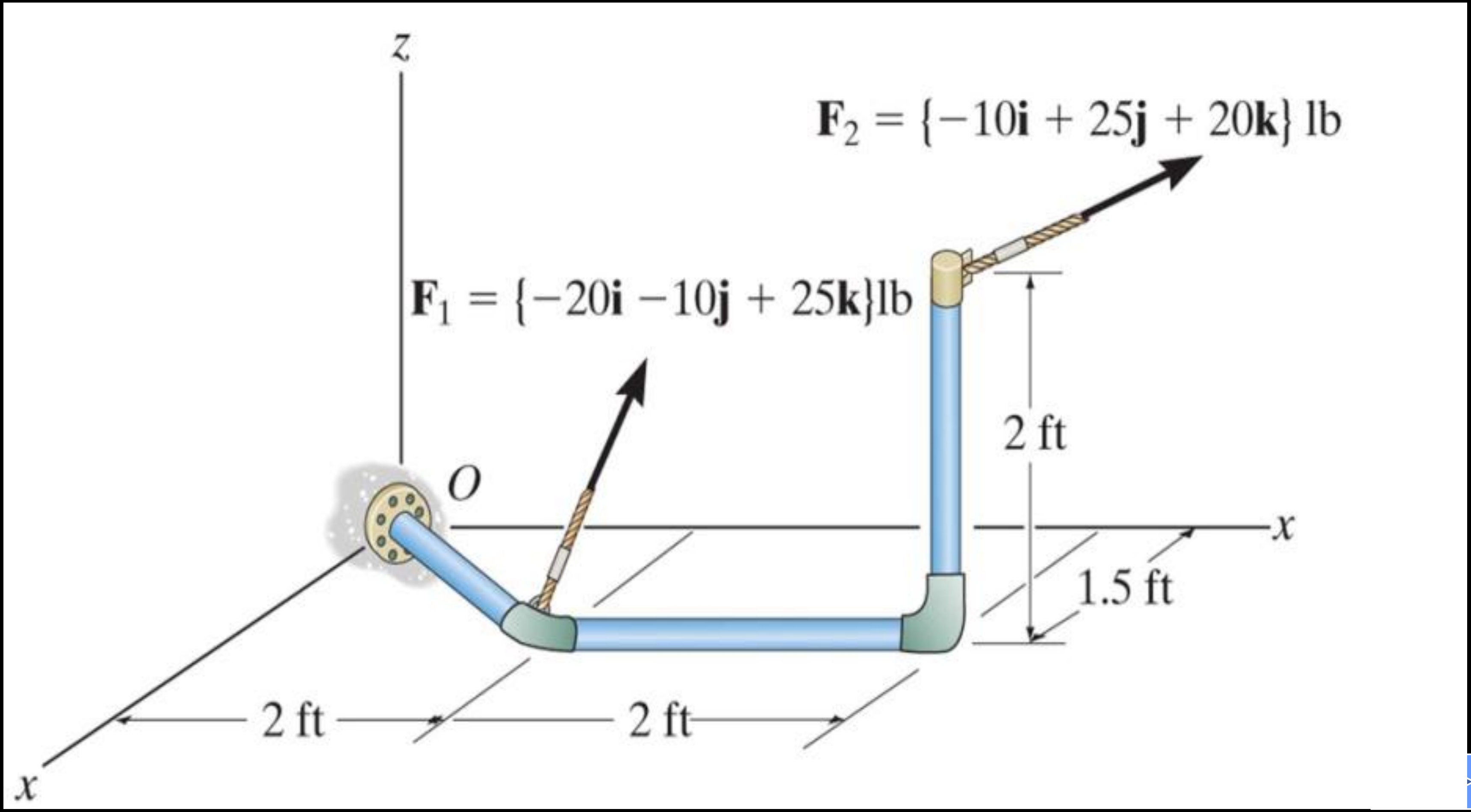
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = P\mathbf{i} + P\mathbf{j} = P(\mathbf{i} + \mathbf{j})$$

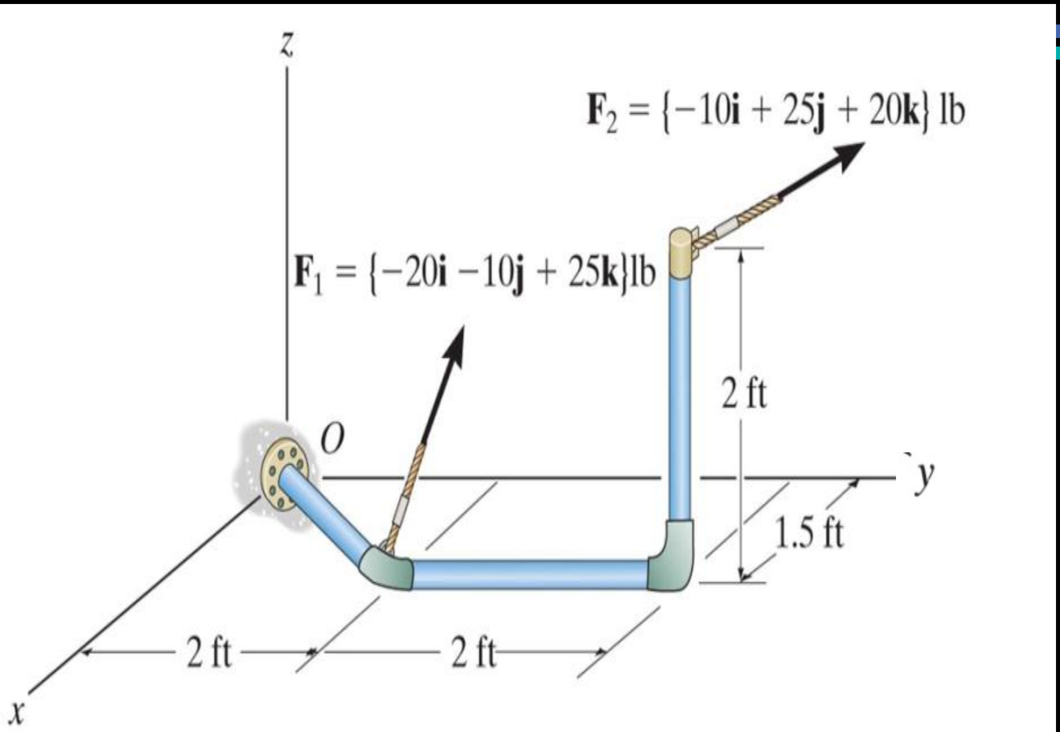
$$\mathbf{M}_O^R = \mathbf{r}_E \times \mathbf{F}_1 + \mathbf{r}_D \times \mathbf{F}_2 = (a\mathbf{i} + a\mathbf{j}) \times P\mathbf{i} + (a\mathbf{j} + a\mathbf{k}) \times P\mathbf{j}$$

$$= -Pa\mathbf{k} - Pa\mathbf{i} = -Pa(\mathbf{i} + \mathbf{k})$$

Given: Forces ***F_1*** and ***F_2*** are applied to the pipe.

Find: An equivalent resultant force and couple moment at point O.





$F_1 = \{-20\,i - 10\,j + 25\,k\}$ lb

$F_2 = \{-10\,i + 25\,j + 20\,k\}$ lb

$F_{RO} = \{-30\,i + 15\,j + 45\,k\}$ lb

$r_1 = \{1.5\,i + 2\,j\}$ ft

$r_2 = \{1.5\,i + 4\,j + 2\,k\}$ ft

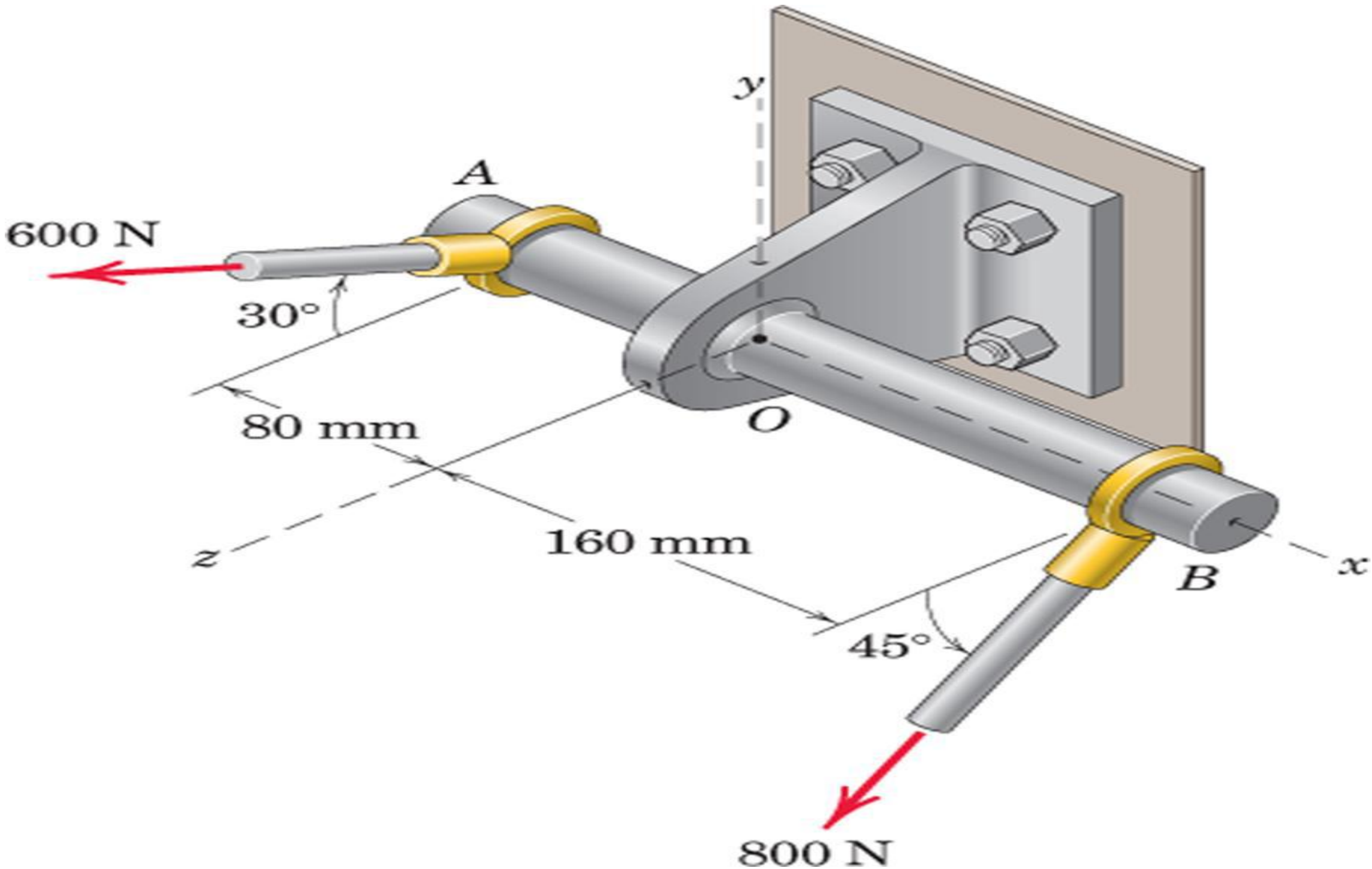
Then, $M_{RO} = \Sigma\,(r_i \times F_i) = r_1 \times F_1 + r_2 \times F_2$

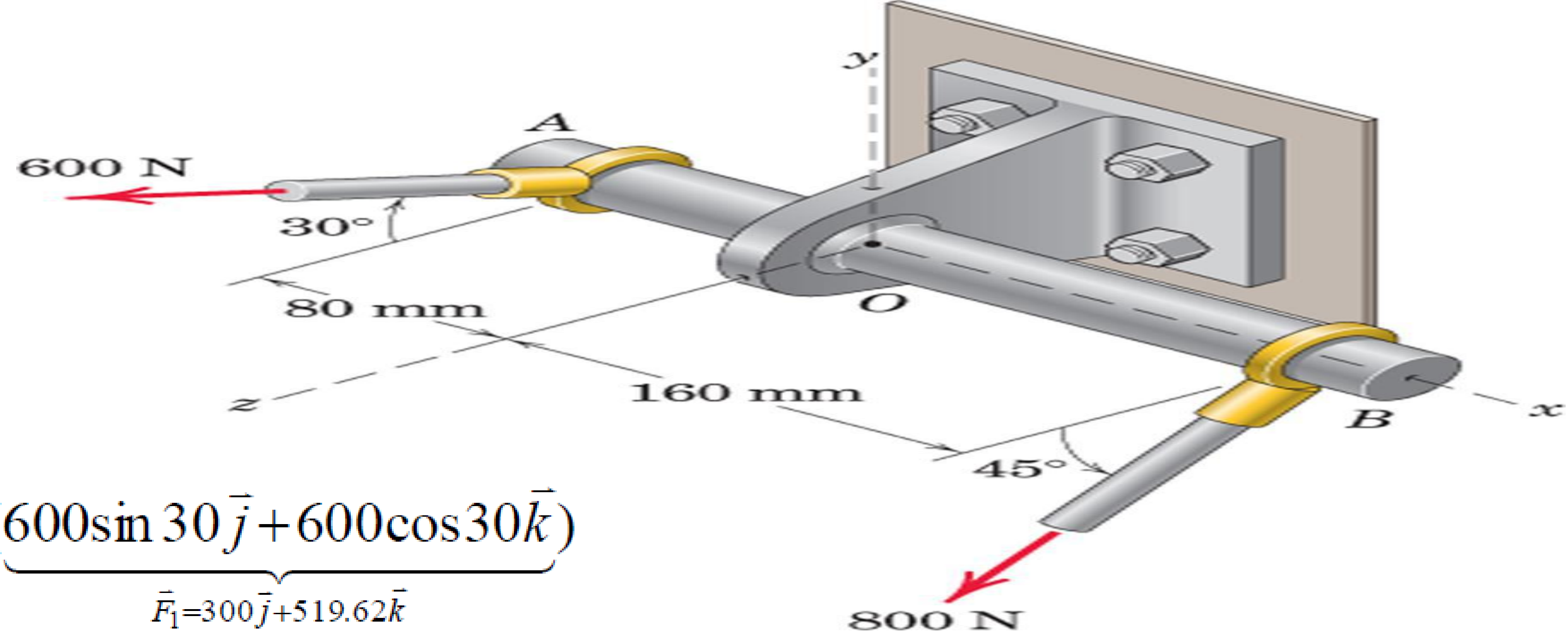
$$M_{RO} = \left\{ \begin{vmatrix} i & j & k \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 1.5 & 4 & 2 \\ -10 & 25 & 20 \end{vmatrix} \right\} \text{ lb}\cdot\text{ft}$$

$$= \{(50\,i - 37.5\,j + 25\,k) + (30\,i - 50\,j + 77.5\,k)\} \text{ lb}\cdot\text{ft}$$

$$= \{80\,i - 87.5\,j + 102.5\,k\} \text{ lb}\cdot\text{ft}$$

Determine the force-couple system at O which is equivalent to the two forces applied to the shaft AOB.





$$\vec{R}=(\underbrace{600\sin 30\vec{j}+600\cos 30\vec{k}}_{\vec{F}_1=300\vec{j}+519.62\vec{k}})$$

$$+(\underbrace{-800\sin 45\vec{j}+800\cos 45\vec{k}}_{\vec{F}_2=-565.69\vec{j}+565.69\vec{k}})$$

$$\vec{R}=-265.69\vec{j}+1085.31\vec{k}\ (N)$$

$$\vec{M}_O=\vec{r}_1\times\vec{F}_1+\vec{r}_2\times\vec{F}_2=-0.08\vec{i}\times\left(300\vec{j}+519.62\vec{k}\right)+0.16\vec{i}\times\left(-565.69\vec{j}+565.69\vec{k}\right)$$

$$\vec{M}_O=-24\vec{k}+41.57\vec{j}-90.49\vec{k}-90.49\vec{j}$$

$$\vec{M}_O=-48.92\vec{j}-114.49\vec{k}\ (N\cdot m)$$

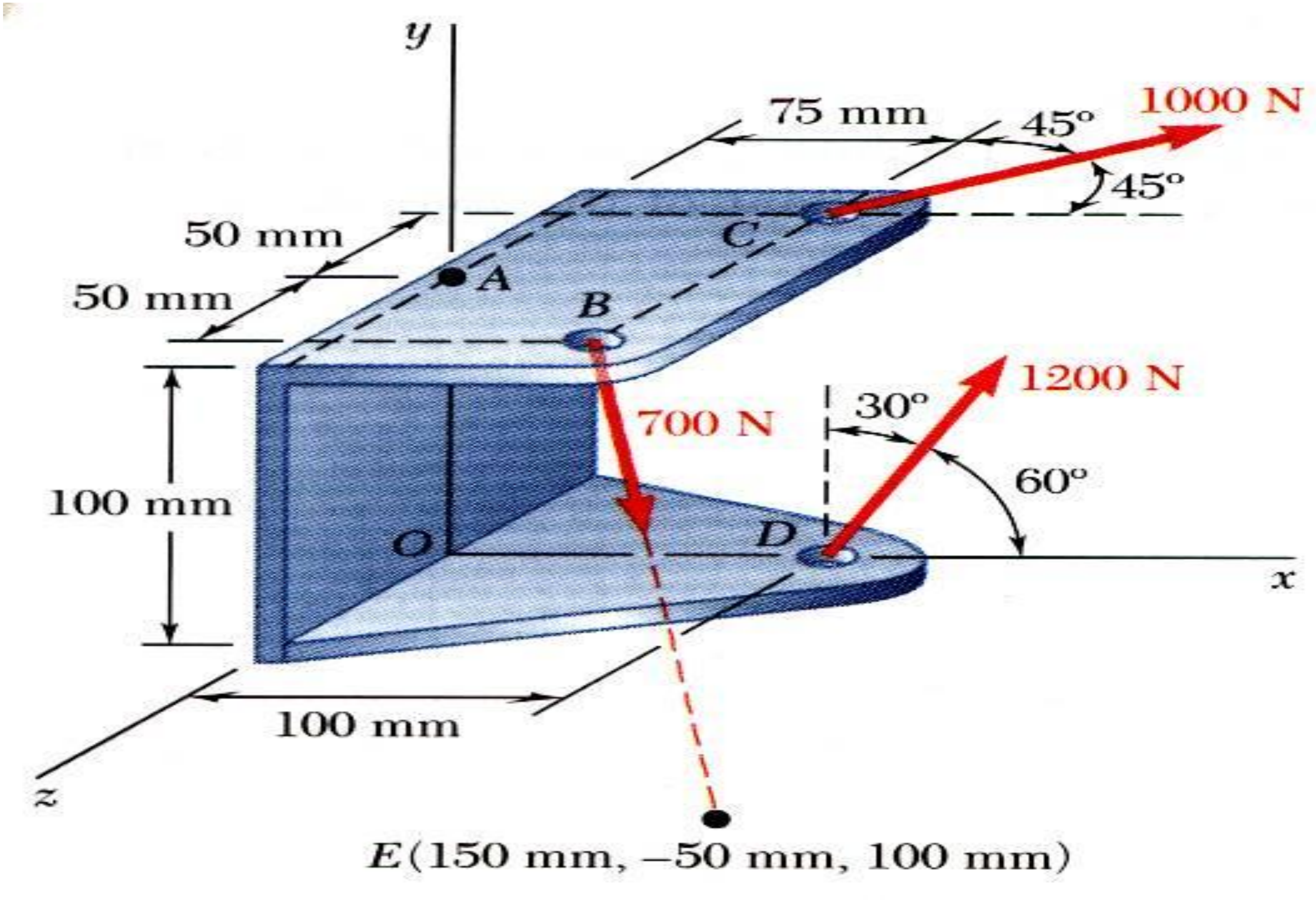
if $\vec{R}\perp\vec{M}_O$, *then* $\vec{R}\cdot\vec{M}_O=0$

$$\vec{R}\cdot\vec{M}_O=(-265.69)(-48.92)+(1085.31)(-114.49)$$

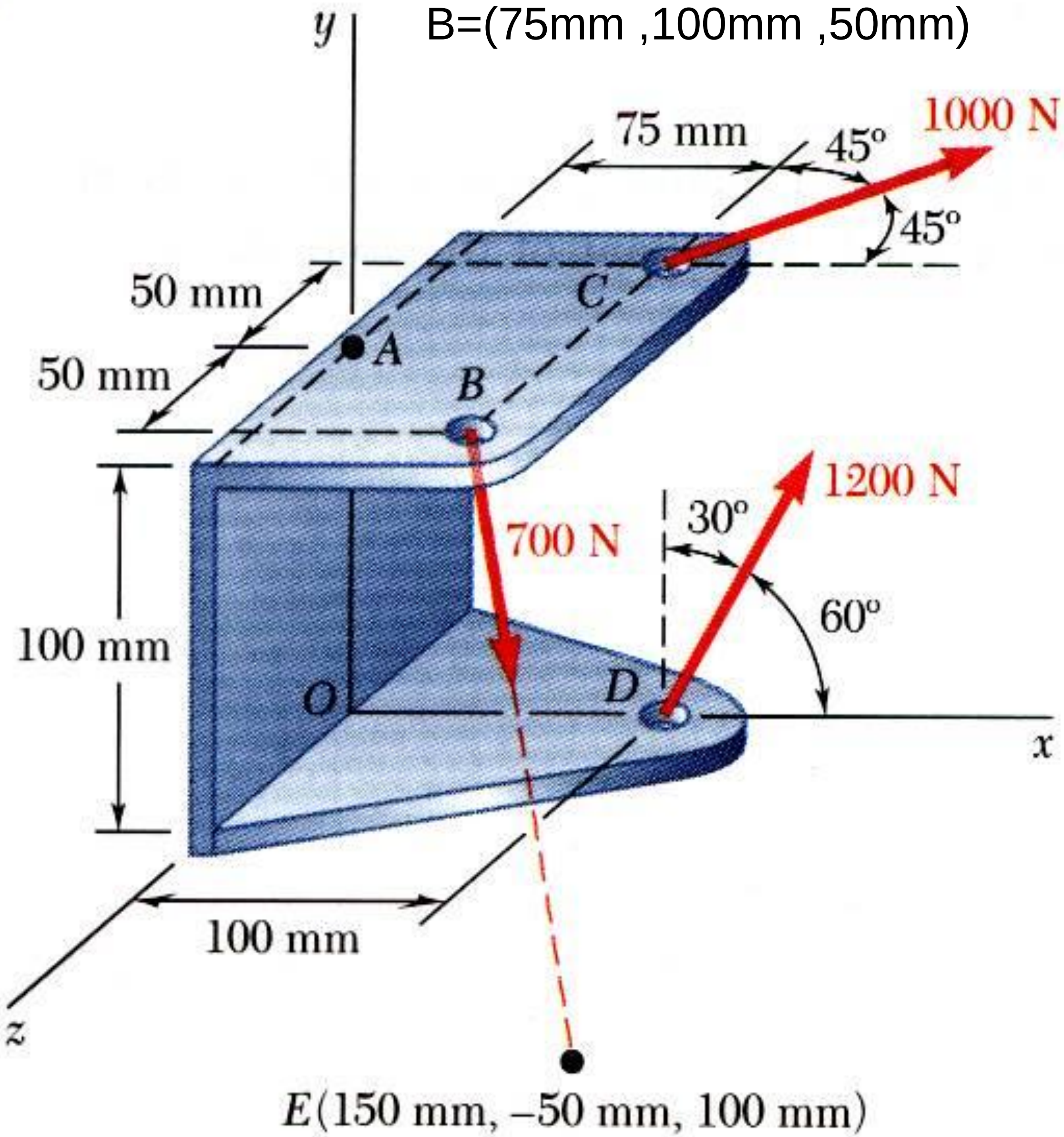
$$=-111260\neq 0$$

they are not perpendicular to each other

- Replace The Forces With An Equivalent Force-Couple System at *A*



B=(75mm ,100mm ,50mm)



$$\vec{F}_B = (700\ \text{N})\hat{u}$$

$$\hat{u} = \frac{\vec{r}_{BE}}{r_{BE}} = \frac{75\vec{i} - 150\vec{j} + 50\vec{k}}{175}$$

$$= 0.429\hat{i} - 0.857\hat{j} + 0.289\hat{k}$$

$$\vec{F}_B = 300\hat{i} - 600\hat{j} + 200\hat{k}\ (\text{N})$$

$$\vec{F}_C = (1000\ \text{N})(\cos 45°\hat{i} - \cos 45°\hat{k})$$

$$= 707\hat{i} - 707\hat{k}\ \ (\text{N})$$

$$\vec{F}_D = (1200\ \text{N})(\cos 60°\hat{i} + \cos 30°\hat{j})$$

$$= 600\hat{i} + 1039\hat{j}\ \ (\text{N})$$

$$\vec{r}_{AB} = 0.075\vec{i} + 0.050\vec{k}\ (\text{m})$$

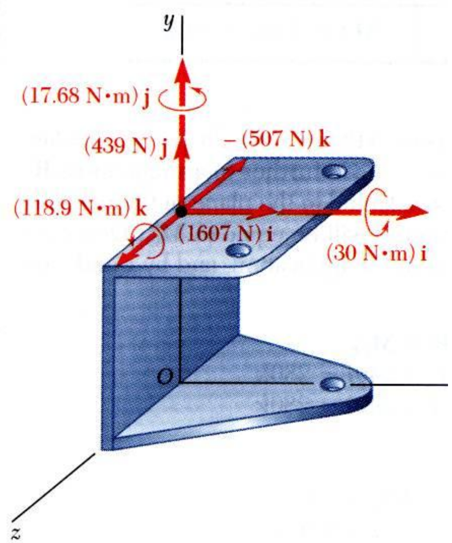
$$\vec{r}_{AC} = 0.075\vec{i} - 0.050\vec{k}\ (\text{m})$$

$$\vec{r}_{AD} = 0.100\vec{i} - 0.100\vec{j}\ (\text{m})$$

- Compute Equivalent Force

$$\vec{R} = \sum \vec{F}$$
$$= (300 + 707 + 600)\hat{i}$$
$$+ (-600 + 1039)\hat{j}$$
$$+ (200 - 707)\hat{k}$$

$$\boxed{\vec{R} = 1607\hat{i} + 439\hat{j} - 507\hat{k} \ \ (\mathrm{N})}$$



- Compute Equivalent Couple

$$\vec{M}_A^R = \sum (\vec{r} \times \vec{F})$$

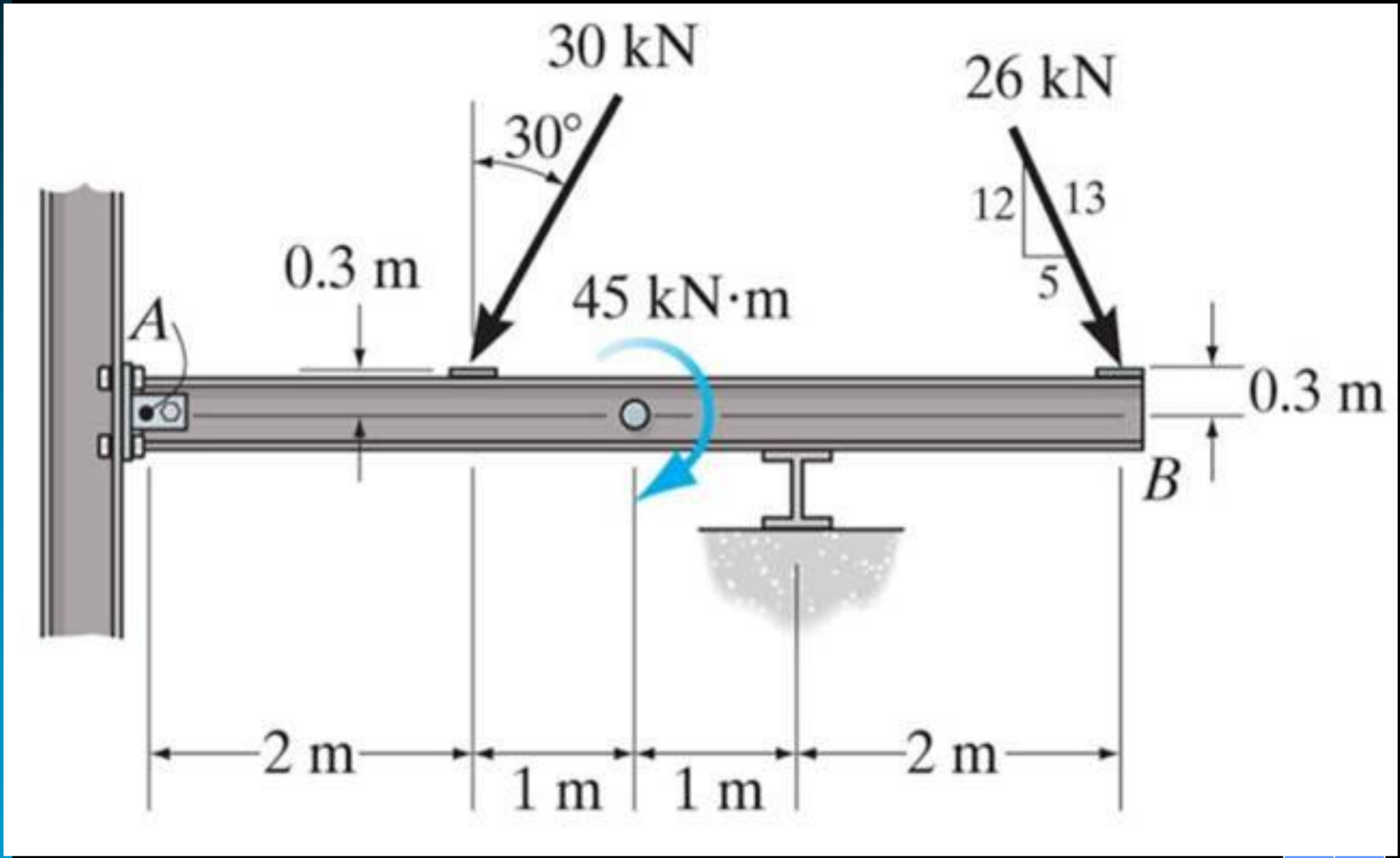
$$\vec{r}_{AB} \times \vec{F}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.075 & 0 & 0.050 \\ 300 & -600 & 200 \end{vmatrix} = 30\hat{i} - 45\hat{k}$$

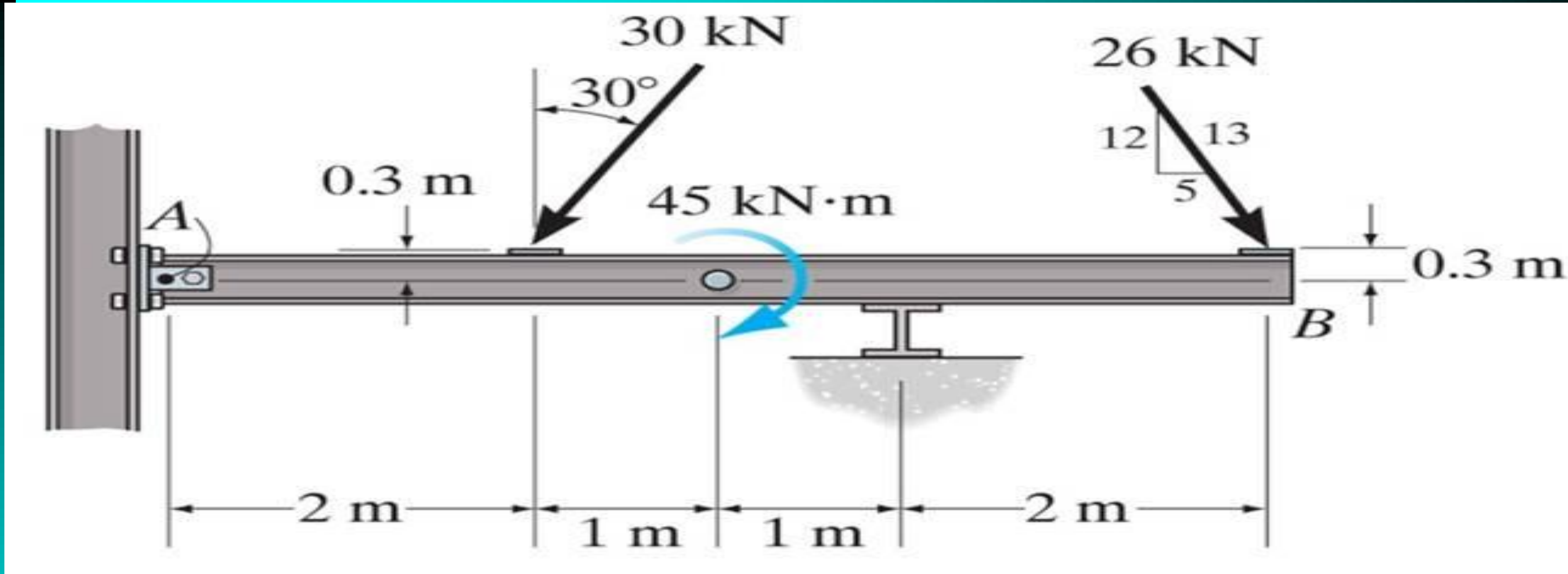
$$\vec{r}_{AC} \times \vec{F}_c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.075 & 0 & -0.050 \\ 707 & 0 & -707 \end{vmatrix} = 17.68\hat{j}$$

$$\vec{r}_{AD} \times \vec{F}_D = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.100 & -0.100 & 0 \\ 600 & 1039 & 0 \end{vmatrix} = 163.9\hat{k}$$

$$\boxed{\vec{M}_A^R = 30\vec{i} + 17.68\vec{j} + 118.9\vec{k}}$$

Given: A 2-D force and couple system as shown. **Find** The equivalent resultant force and couple moment acting at A.





$+\rightarrow \Sigma F_x = (5/13)\,26 - 30 \sin 30° = -5$ kN

$+\uparrow \Sigma F_y = -(12/13)\,26 - 30 \cos 30° = -49.98$ kN

Now find the magnitude and direction of the resultant.

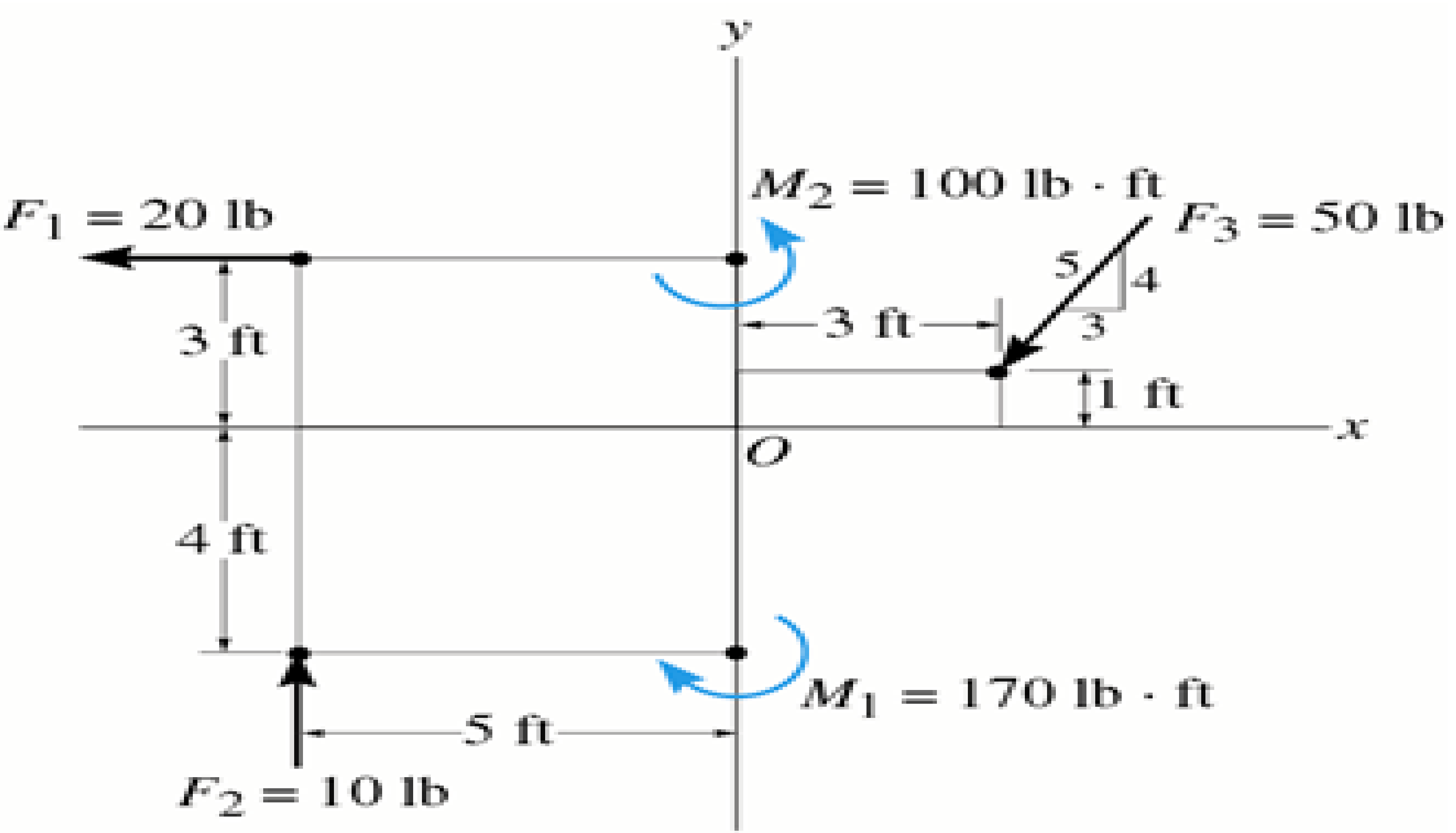
$F_{RA} = (5^2 + 49.98^2)^{1/2} = 50.2$ kN and $\theta = \tan^{-1}(49.98/5)$

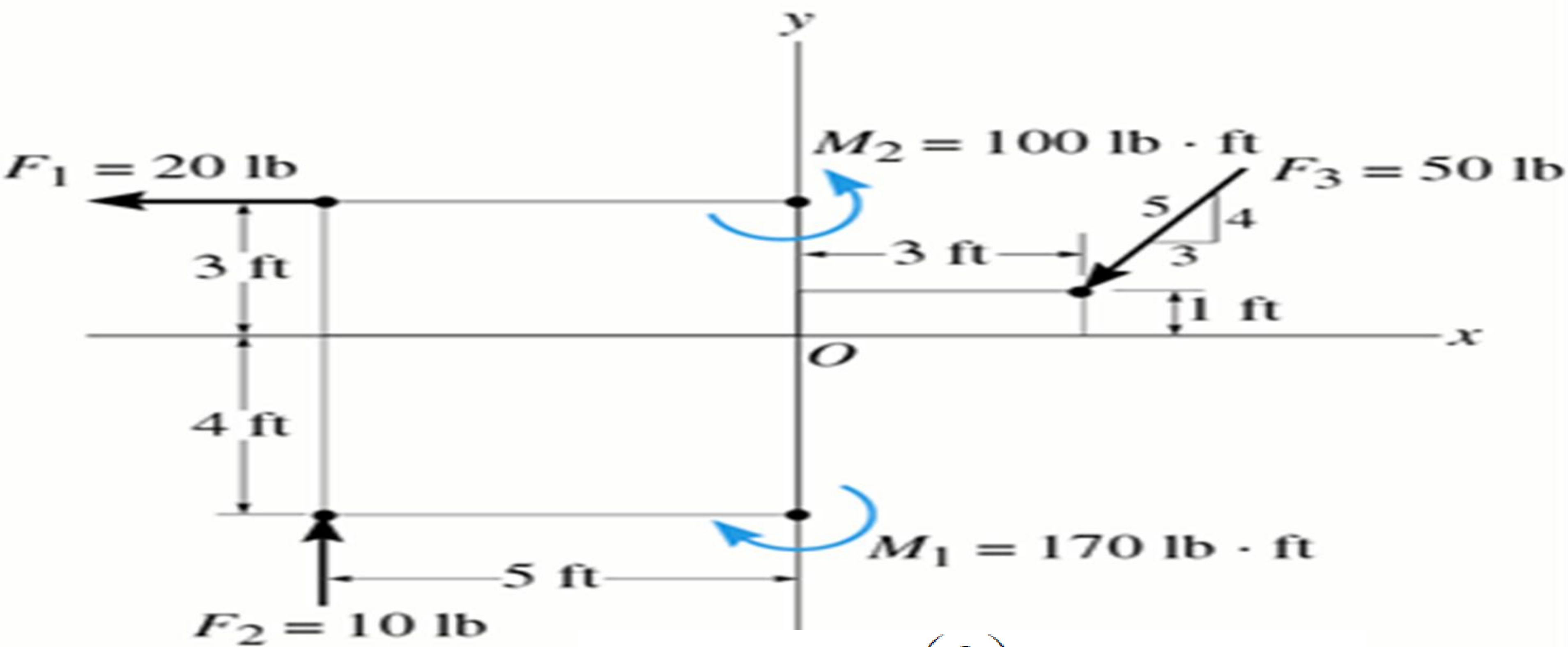
$= 84.3°$ ↙

$+\circlearrowright M_{RA} = \{30 \sin 30° (0.3m) - 30 \cos 30° (2m) - (5/13)\,26\,(0.3m)$

$- (12/13)\,26\,(6m) - 45\} = -239$ kN m

Replace the system by a force and couple-moment at point o .





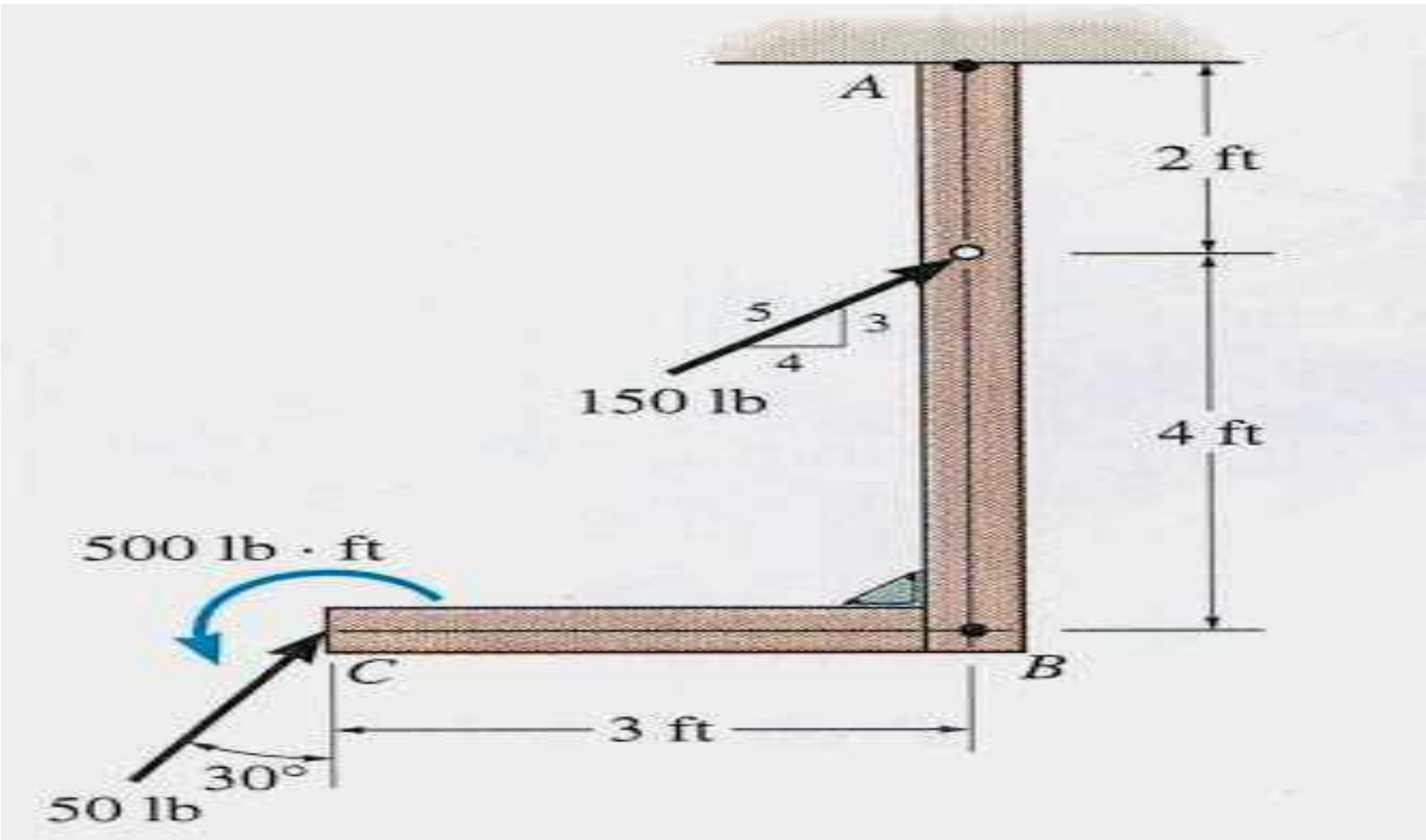
$$\xrightarrow{+} \sum F_{R_x} = \sum F_x \qquad \Rightarrow F_{R_x} = -50\left(\frac{3}{5}\right) - 20 = -50 \text{ lb}$$

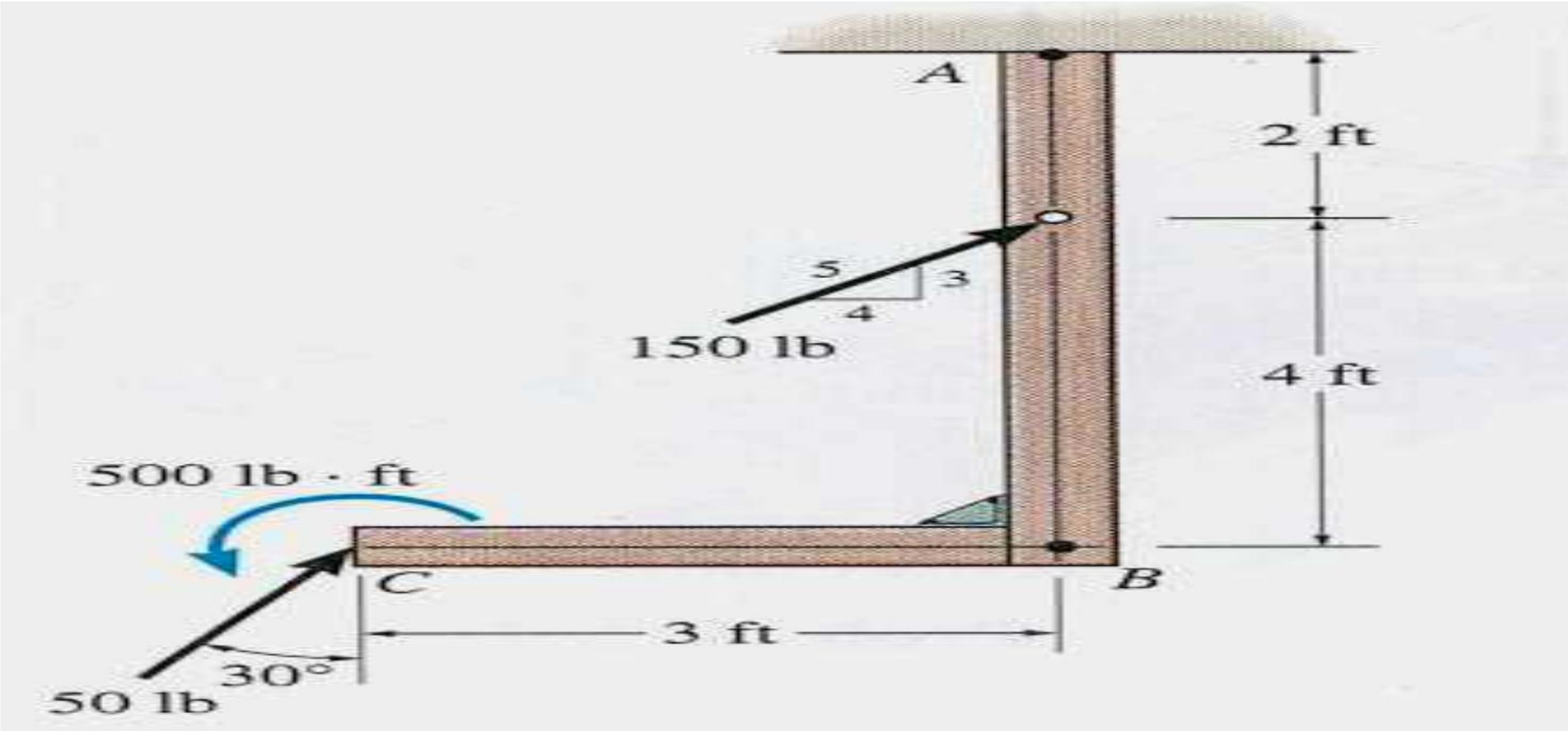
$$+\uparrow \sum F_{R_y} = \sum F_y \qquad \Rightarrow F_{R_y} = -50\left(\frac{4}{5}\right) + 10 = -30 \text{ lb}$$

$$F_R = \sqrt{(-50)^2 + (-30)^2} = 58.3 \text{ lb} \qquad \theta = \tan^{-1}\left(\frac{-30}{-50}\right) = 31.0°$$

$$\circlearrowleft + M_{R_O} = \sum M_O = -50\left(\frac{4}{5}\right)(3) + 50\left(\frac{3}{5}\right)(1) + (20)(3) + 100 - 170 - 10(5) = -150 \text{ lb.ft}$$

Given:A 2-D force and couple system as shown Find: The equivalent resultant force and couple moment acting at A





$+\rightarrow \Sigma F_x = (4/5)\ 150\ \text{lb} + 50\ \text{lb} \sin 30° = 145\ \text{lb}$

$+\uparrow \Sigma F_y = (3/5)\ 150\ \text{lb} + 50\ \text{lb} \cos 30° = 133\ \text{lb}$

$F_{RA} = (145^2 + 133.3^2)^{1/2} = 197\ \text{lb}$

$\theta = \tan^{-1}(133.3/145)$
$= \mathbf{42.6°}$ ∠

$+\circlearrowright M_{RA} = \{ (4/5)(150)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500 \} = 760\ \text{lb·ft}$